Grade 6 Math Unit 3 - Expressions and Equations

UNIT OVERVIEW

In grade 6, instructional time should focus on four critical areas. This unit addresses **Critical Focus Area #3, Writing, interpreting and using expressions and equations.** (See Connections for explanation)

This unit will address the following clusters:
- Apply and extend previous understanding of arithmetic to algebraic expressions
- Reason about and solve one-variable equations and inequalities

Students will understand real-world application of inequalities and be able to graph on a number line. Also, students will examine two variable equations, understand the concepts of independent and dependent variables, and be able to make a table where they will pair the data and graph the ordered pairs on a coordinate plane.

STANDARDS

CC_Common Core State Standards - Mathematics (2010) - Grade 6
Domain 6.EE Expressions and Equations
Cluster Statement: Apply and extend previous understandings of arithmetic to algebraic expressions.
Standard 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.
Standard 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
6.EE.2.a Write expressions that record operations with numbers and with letters standing for numbers.
6.EE.2.b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
6.EE.2.c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
Standard 6.EE.3 Apply the properties of operations to generate equivalent expressions.
Standard 6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Cluster Statement: Reason about and solve one-variable equations and inequalities.
Standard 6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
Standard 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
Standard 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
Standard 6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Cluster Statement: Represent and analyze quantitative relationships between dependent and independent variables.

Standard 6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

CONTENT ELABORATIONS

Work in this unit will build upon prior knowledge of **6.NS.3**, **6.NS.4** and **6.RP.3**

6.EE.1

6.EE.1 Students demonstrate the meaning of exponents to write and evaluate numerical expression with whole number exponents. The base can be a whole number, positive decimal or a positive fraction. Students recognize that an expression with a variable represents the same mathematics and write algebraic expressions from verbal expressions.
Examples: Write the following as a numerical expressions using exponential notation.

- the area of a square with a side length of 8m (8^2m^2)
- the volume of a cube with a side length of 5 ft. (5^3 ft^3)
- Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own. (2^3 mice)

MP.2 should be emphasized.

6.EE.2a-c

6.EE.2a-c Students write expressions from verbal descriptions using letters and numbers. Students understand order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, n" could be represented with 5n and that a number and letter written together means to multiply.
Students use appropriate mathematical language to write verbal expressions from algebraic expressions. Students can describe expressions such as 3 (2 +6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms.
Students evaluate algebraic expressions, using order of operations as needed. Given an expression such as 3x + 2y, find the value of the expression when x is equal to 4 and y is equal to 2.4. This problem requires students to understand that multiplication is understood when numbers and variables are written together and to usse the order of operations to evaluate. 3 x 4 + 2 x 2.4 = 12 + 4.8 = 16.8.
Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. For example, it costs $100 to rent the skating rink plus $5 per person. The cost for any number (n) of people could be found by the expression, 100 + 5n. What is the cost for 25 people?
It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

- r + 21 as "some number plus 21 as well as "r + 21"
- n x 6 as "some number times 6 as well as "n times 6"

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.
Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.
Consider the following expression: x^2 + 5y + 3x + 6

- The variables are x and y
- There are 4 terms, x^2, 5y, 3x, and 6
- There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively.
- There is one constant term, 6.
- The expression shows a sum of all four terms.

Examples:

- 7 more than 3 times a number; 3x + 7
- 3 times the sum of a number and 5; 3(x+5)
- 7 less than the product of 2 and a number; 2x - 7

6.EE.3	**6.EE.3** Students use the distributive property to write equivalent expressions. Area models from elementary grades can be used to illustrate the distributive property with variables. Examples: - The expression 10x + 15 could be represented in an area model. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors of the area model would be 5(2x + 3) - Students use their understanding of multiplication to interpret 3(2 +x). For example, 3 groups of (2 + x). They use a model to represent x, and make an array to show the meaning of 3(2 + x). They can explain why it makes sensse that 3(2 + x) is equal to 6 + 3x. (An array with 3 columns and x + 2 in each column). **MP.2, MP.3, MP.4, MP.6, MP.7** should be emphasized.
6.EE.4	**6.EE.4** Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distraibutive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form. **MP.2, MP.3, MP.4, MP.6, MP.7** should be emphasized.
6.EE.5	Beginning experiences in solving equations should require students to understand the meaning of the equation as well as the question being asked. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies such as using reasoning, fact families, and inverse operations. Students may use balance models in representing and solving equations and inequalities. Consider the following situation: Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him? This situation can be represented by the equation 26 + n = 100 where n is th enumber of papers the tacher gives Joey. This equation can be stated as "some number was added to 26 and the result was 100". Students ask themselves "What number was added to 26 to get 100?" to help them detemine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem. - Reasoning: 26 + 70 is 96. 96 + 4 is 100, so the number added to 26 to get 100 is 74. - Use knowledge of fact families to write related equations: n + 26 = 100, 100 - n = 26, 100 - 26 = n. Select the equation that helps you find n easily. - Use knowledge of inverse operations: Since subtraction "undoes" addition then subtract 26 from 100 to get the numerical value of n. - Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance. - bar model strategy *Examples:* - The equation 0.44s = 11 where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies you used to determine your answer. show that your solution is correct using substitution. - Twelve is less than 3 times another number can be shown by the inequality 12 < 3n. What numbers could possibly make this a true statement? Students identify values from a specified set that will make an equation true. For example, given the expression x + 2 ½ which of the following value(s) for x would make x + 2 ½ = 6. {0, 3 ½, 4} By using substitution, students identify 3 ½ as the value that will make both sides of the equation equal. The solving of inequalities is limited to choosing values from a specified set that would make the inequality true. Students use substitution to identify the values to make the inequality true. **MP.1, MP.2, MP.4, MP.7** should be emphasized.

6.EE.6	**6.EE.6** Students write expressions to represent various real-world situations. For example, the expression a + 3 could represent Susan's age in three years, when a represents her present age. The expression 2n represents the number of wheels on any number of bicycles. Conncecting writing expressions with story problems and /or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforcees that the variable represents a number. Examples: - Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. (Solution: 2c + 3 where c represents the number of crayons that Elizabeth has.) - Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He has paid $85 for completing one yard. Write an equation to repressent theamoind of money he earned. (Solution: 15h + 20 = 85 where h is the number of hours worked.) **MP.2, MP.3, MP.4, MP.6, MP.7** should be emphasized.
6.EE.7	Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is know. For example, in the expression, x + 4, any value can be substituted for the x to generate a numerical answer; however, in the equation x + 4 = 6, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable. Students write equations from real-world problems and then use inverse operations to solve one-step equations. Equations may include fractions and decimals with non-negative solutions. Students create and solve equations that are based on real-word situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies. *Example:* Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost. *Solution* : Students might say, " I created a bar model to show the cost of the three pair of jeans. Each bar labeled j is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation 3j = $56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 = 30 but less than $20 each because 20 x 3 = 60. If I start with $15 each, I end up with $45. I have $11.58 left. I then give each pair of jeans $3. That's 9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15 + 3 + 0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58". **MP.1, MP.2, MP.3, MP.4, MP.7** should be emphasized.
6.EE.8	Many real-world situations are represented by inequalities. Students write an inequality and represent solutions on a number line for various contextual situations. For example, the class must raise at least $80 to go on a field trip. If m represents money, then the inequality m > to $80. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading. A number line diagram is drawn with an open circle when an inequality contains < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions. **MP.1, MP.2, MP.3, MP.4, MP.7** should be emphasized.

6.EE.9	The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis. Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fraction parts such as people, tents, records etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would no be considered. A line is drawn when both variables could be represented with fractional parts. Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) Relationships should be proportional with the line passing through the origin. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and/or a table of values. Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function. **MP.1, MP.2, MP.3, MP.4, MP.7, MP.8** should be emphasized.

UNIT VOCABULARY

algebra	defining the variable	like terms
algebraic expression	Distributive Property	numerical expression
Associative Properties	equivalent expressions	perfect square
base	evaluate	powers
coefficient	exponent	properties
Commutative Properties	factor the expression	term
constant	Identity Properties	variable
Addition Property of Equality	Multiplication Property of Equality	function
Division Property of Equality	solution	function rule
equals sign	solve	function table
equation	Subtraction Property of equality	geometric sequence
expressions	arithmetic sequence	independent variable
inverse operations	dependent variable	inequality

BIG IDEAS

ENDURING UNDERSTANDINGS	ESSENTIALS QUESTIONS Choose a few questions based on the needs of your students

•Variables can be used as unique unknown values or as quantities that vary.
•Exponential notation is a way to express repeated products of the same number.
•Algebraic expressions may be used to represent and generalize mathematical problems and real life situations
•Properties of numbers can be used to simplify and evaluate expressions.
•Algebraic properties can be used to create equivalent expressions
•Two equivalent expressions form an equation.
• Different forms of representation can be used to show a relationship.
*Values from specified sets are used to make an equation or inequality true.
•Graphs can be used to represent all of the possible solutions to a given situation.
•Many problems encountered in everyday life can be solved using proportions, equations or
inequalities.

•How are "standard form" and "exponential form" related?
•What is the purpose of an exponent?
•How are exponents used when evaluating expressions?
•How is the order of operations used to evaluate expressions?
•How are exponents useful in solving mathematical and real world problems?
How are properties of numbers helpful in evaluating expressions?
•What strategies can I use to help me understand and represent real situations using algebraic expressions?
•How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
•How is the Distributive Property used to evaluate, simplify and expand expressions?
•How can I tell if two expressions are equivalent?
•How is an equation like a balance? How can the idea of balance help me solve an equation?
•What strategies can I use to help me understand and represent real situations using proportions, equations and inequalities?
•How can I write, interpret and manipulate proportions, equations, and inequalities?
•How can I solve a proportion and an equation?
•How can I tell the difference between an expression, equation and an inequality?
•How are the solutions of equations and inequalities different?
•What does an equal sign mean mathematically?
•How can proportions be used to solve problems?

CONNECTIONS

In Critical Focus Area #3, students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as 3x = y) to describe relationships between quantities.

The learning in this unit's clusters is foundational in the transition to algebraic representation and problem solving which is extended and formalized in Grade 7, the Number System and Expressions and Equations.

Standards for Mathematical Practice (SMP)
MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure (Deductive reasoning)
MP.8 Look for and express regularity in repeated reasoning (Inductive Reasoning)

APPLY AND EXTEND PREVIOUS UNDERSTANDING OF ARITHMETIC TO ALGEBRAIC EXPRESSIONS

	CONTENT	SKILLS
6.EE.1	Write and evaluate numerical expressions involving whole-number exponents	Write and evaluate numerical expressions involving whole-number exponents 1. Write numerical expressions involving whole-number exponents 2. Evaluate (solve) numerical expressions involving whole-number exponents 3. Solve order of operation problems that contain exponents
6.EE.2	Write, read and evaluate expressions in which letters stand for numbers	Write, read and evaluate expressions in which letters stand for numbers 1. Use numbers and variables to represent desired operations 2. Translate written phrases into algebraic expressions. 3. Translate algebraic expressions into written phrases. 4. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) 5. Identify parts of an expression as a single entity, even if not a monomial. 6. Substitute specific values for variables 7. Evaluate algebraic expressions, including those that arise from real-world problems 8. Apply order of operations when there are no parentheses for expressions that include whole number exponents
6.EE.3	Apply the properties of operations to generate equivalent expressions.	Apply the properties of operations to generate equivalent expressions. 1. Generate equivalent expressions using the properties of operations. (e.g. distributive property, associative property, adding like terms with the addition property of equality, etc.)
6.EE.4	Identify when two expressions are equivalent	Identify when two expressions are equivalent 1. Recognize when two expressions are equivalent. 2. Prove (using various strategies) that two equations are equivalent no matter what number is substituted.

REASON ABOUT AND SOLVE ONE-VARIABLE EQUATIONS AND INEQUALITIES

	CONTENT	SKILLS
6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true?	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? 1. Recognize solving an equation or inequality as a process of answering “which values from a specified set, if any, make the equation or inequality true?” 2. Know that the solutions of an equation or inequality are the values that make the equation or inequality true. 3. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem.	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. 1. Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 2. Relate variables to a context. 3. Write expressions when solving a real-world or mathematical problem.
6.EE.7	Solve real-world and mathematical problems by writing and solving equations.	Solve real-world and mathematical problems by writing and solving equations. 1. Understand what a solution is, and use substitution to verify solutions to equations. 2. Define inverse operation. 3. Know how inverse operations can be used in solving one-variable equations. 4. Apply rules of the form x+p=q and px=q, for cases in which p, q and x are all nonnegative rational numbers, to solve real world and mathematical problems. (There is only one unknown quantity.) 5. Develop a rule for solving one-step equations using inverse operations with nonnegative rational coefficients. 6. Solve and write equations for real-world mathematical problems containing one unknown.
6.EE.8	Examine variable inequalities.	Examine variable inequalities. 1. dentify the constraint or condition in a real-world or mathematical problem in order to set up an inequality. 2. Recognize that inequalities of the form x>c or x<c have infinitely many solutions. 3. Write an inequality of the form x>c or x<c to represent a constraint or condition in a real-world or mathematical problem. 4. Represent solutions to inequalities or the form x>c or x<c, with infinitely many solutions, on number line diagrams. 5. Use substitution to check if specific numbers are solutions
REPRESENT AND ANALYZE QUANTITATIVE RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES		
	CONTENT	**SKILLS**
6.EE.9	Write and examine equations with dependent and independent variables.	Write and examine equations with dependent and independent variables. 1. Understand that a variable can represent one unknown number or a set of numbers (ie. multiple variables, dependent variables) 2. Define independent and dependent variables. 3. Use variables to represent two quantities in a real-world problem that change in relationship to one another. 4. Write an equation to express one quantity (dependent) in terms of the other quantity (independent). 5. Analyze the relationship between the dependent variable and independent variable using tables and graphs. 6. Relate the data in a graph and table to the corresponding equation.
UNIT RESOURCES		

Common Core Model Curriculum
McGraw-Hill, **Glencoe Math**, Chapter 6-8
Georgia Math frameworks, Grade 6, Unit 3 & Unit 4
Manipulatives
Smart Board resources
Hands-On Standards